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RESEARCH ARTICLE

Some Considerations on Solving Trigonometric Equations Involving the Absolute Value Sign

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Abstract

The article is devoted to the study of trigonometric equations taught in general secondary schools and which students encounter some difficulties in mastering. It describes trigonometric equations involving the modulus sign and methods for solving them. The article also explains the modulus of a real number and its properties using examples.

KEYWORDS

Modulus of a real number, modular equation, trigonometric equation, general solution, modular expression, modulus of addition, modulus of multiplication, modulus of division.

INTRODUCTION

It is known that in recent years, great importance has been attached to a radical reform of the education system in our Republic, including mathematical education. A vivid example of this is the Presidential Decree on measures to further develop mathematical education and sciences in our Republic. The tasks set out in this decree also directly relate to the level of general secondary schools, which are considered one of the important links in the continuous education system. Because the groundwork for training highly qualified, competitive, and modern specialists is mainly created in general secondary schools. Therefore, great importance should be attached to teaching mathematics at this stage. As is known, one of the meaningful methodological directions of the mathematics course in general secondary schools is equations. Because equations and related concepts are studied today starting from the first grade of general secondary schools. Among these equations, trigonometric equations involving the modulus sign have a special place. Because when solving such equations, students first use the modulus of a real number and its properties, and secondly, they use the concept of a trigonometric equation and methods for solving it. This is important for further developing students' creative thinking skills.

The purpose of the article is to show effective methods of teaching trigonometric equations involving the modulus sign in general secondary schools and to provide methodological recommendations for the formation of students' skills and competencies in solving such equations. The main objectives of the article are: to explain the modulus of a real number and its properties using examples; to show various methods of solving trigonometric equations involving the modulus sign; to consolidate theoretical knowledge using examples of varying complexity.

It is also worth noting that modular trigonometric equations are widely used not only in the school mathematics curriculum, but also in international mathematical olympiads, and in exam tasks of the State Testing Center (DTM). Therefore, in-depth study of this topic directly contributes to the competitiveness of students.

In the field of real numbers, one of the most characteristic concepts of a number is the concept of its modulus. This concept is used not only in solving equations, but also in various areas of mathematics, including the fundamental concepts of mathematical analysis, in defining the limit of a calculated function and the limit of a numerical sequence. In addition, it is used in defining the concept of the absolute error of an approximate number, in characterizing the concept of a vector in mechanics, in formulating rules for performing operations on rational numbers, in the process of studying the arithmetic square root of a number and its properties, in the topic of rational exponents, and in studying the properties of n-th roots.

There are various definitions of the modulus of a real number that are equally strong. Below we present some of them.

Definition 1. The modulus of $|a|$ *a real number a* is called the number denoted by and defined as:

$$|a| = \begin{cases} a, & agar\ a > 0\ bo'lsa, \\ 0, & agar\ a = 0\ bo'lsa, \\ -a, & agar\ a < 0\ bo'lsa \end{cases}$$

Definition 2. The modulus of a real number $M(a)$ *a* is the distance from a point to the origin and $|a|$ is defined as

Definition 3. The modulus of a real number *a is the positive square root of the product of a* itself and *its* opposite, and is expressed by the following formula: $|a| = \sqrt{a^2}$.

This definition is convenient when calculating modulus algebraically, and is especially widely used in simplifying expressions and proving inequalities.

Geometric interpretation of modulus: On the number line, the number $|a|$ represents the distance from point *a* to point 0 (zero). For example, $|-3| = 3$ and $|3| = 3$, since points −3 and 3 are both 3 units from zero. Therefore, the modulus is never negative: $|a| \geq 0$, for all real numbers *a* .

Graph of the function *y* = | *x |: The function y* = | *x* | has the following form: when *x* ≥ 0 , *y* = *x* (first quadrant), when *x* < 0 , *y* = −*x* (second quadrant). The graph starts from the positive part of the coordinate axis and is located in the shape of the letter *V* , with its tip located at the origin of the coordinates — (0, 0). This function is an even function, and its graph is symmetric about *the Y* axis.

Modular inequalities: It is also useful to use modular inequalities when solving modular equations. The main forms are: The inequality | *f(x)* | < *a* (*a > 0)* is as strong as *a < f(x) < a ; the inequality | f(x)* | > *a* (*a* > 0) is as strong as *f(x)* < −*a* or *f(x)* > *a* . Knowing these rules is a great help in solving complex trigonometric equations involving the module sign.

For example, $|17| = 17, \quad |-9| = -(-9) = 9, \quad |0| = 0.$

Example 1. $a = -3, \; b = 5$ Find the value of if . $|a| + |2b|$

Solution. $|a| + |2b| = |-3| + |2 \cdot 5| = -(-3) + 10 = 3 + 10 = 13.$

Example 2: If $a = 1, \; b = 2$, $\left(-|-a|\right)^3 + 2|-b|^3 - |a| \cdot b$ calculate the value of.

Solution. $\left(-|-a|\right)^3 + 2|-b|^3 - |a| \cdot b = \left(-|-1|\right)^3 + 2 \cdot |-2|^3 - |1| \cdot 2 = -1 + 16 - 2 = 13$

The modulus of a real number has a number of properties:

1^0. The modules of opposite numbers are equal to each other, that is,

$$|a| = |-a|$$

2^0. The modulus of the sum of finitely many real numbers is not greater than the sum of the moduli of the adders, i.e.

$$|a_1 + a_2 + \dots + a_n| \le |a_1| + |a_2| + \dots + |a_n|$$

3^0. Two real number difference module decreasing and parting numbers modules from the assembly big be ca n't , that is

$$|a - b| \le |a| + |b|$$

4^0. The modulus of the difference or sum of two numbers is not less than the modulus of the difference of the moduli of these numbers and not greater than the sum of the moduli of these numbers, i.e.

$$||a| - |b|| \le |a \pm b| \le |a| + |b|$$

5^0. The modulus of the product of a finite number of factors is equal to the product of the moduli of the factors, i.e.

$$|a_1 \cdot a_2 \cdot \ldots \cdot a_n| = |a_1| \cdot |a_2| \cdot \ldots \cdot |a_n|$$

6^0. The modulus of the division of two numbers is equal to the modulus of the divisor divided by the modulus of the divisor (when the divisor is different from zero), i.e.

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|},\ b \ne 0.$$

These properties are used when simplifying modular expressions, solving modular equations and inequalities. Therefore, students need to master these properties well.

Example 3. $|x-3| + |x+4|$ Write the expression without the modulus symbol.

Solution. To write this expression without the modulus symbol, first $x-3$ and $x+4$ We find the intervals of each of the expressions in which the signs do not change. Such intervals $x=3$ and $x=-4$ are created using points. They $(-\infty; -4)$, $[-4; 3)$ and $[3; +\infty)$ consists of intervals.

1) $x \in (-\infty; -4)$ for Since $x-3<0$ and $x+4<0$ is $|x-3| = -(x-3) = 3-x$ and $|x+4| = -(x+4) = -x-4$ is , in this case the expression given is equal to:

$$|x-3| + |x+4| = 3 - x - x - 4 = -2x - 1;$$

2) $x \in [-4; 3)$ for Since $x-3<0$ and $x+4>0$ is $|x-3| = -(x-3) = 3-x$ and $|x+4| = x+4$ is , in this case the given expression is equal to:

$$|x-3| + |x+4| = 3 - x + x + 4 = 7;$$

3) $x \in [3; +\infty)$ for Since $x-3>0$ and $x+4>0$ is $|x-3| = x-3$ and $|x+4| = x+4$ is , in this case the given expression is equal to:

$|x-3|+|x+4| = x-3+x+4 = 2x+1$.

And so,

$$|x-3|+|x+4| = \begin{cases} -2x-1, & agar\ x<-4 \quad bo'lsa, \\ 7, & agar\ -4 \le x < 3 \quad bo'lsa, \\ 2x+1, & agar \quad x \ge 3 \quad bo'lsa. \end{cases}$$

To solve equations involving modulus, the given equation is written without the modulus symbol and replaced by a set of equations of equal strength. The following are often used to solve equations involving modulus:

1) $|f(x)| = a \quad (a \ge 0)$ equations $f(x) = a$ and $f(x) = -a$ equations in the form are brought to solution;

2) $|f(x)| = \varphi(x)$ equations $\begin{cases} f(x) = \varphi(x), \\ \varphi(x) \ge 0 \end{cases}$ and $\begin{cases} f(x) = -\varphi(x), \\ \varphi(x) \ge 0 \end{cases}$ systems of the form;

3) $|f(x)| = |g(x)|$ The equation $f(x) = g(x)$ and equations $f(x) = -g(x)$ in the form are brought to solution.

In practice $f(x)$, $\varphi(x)$ and $g(x)$ The functions can be linear, quadratic, exponential, logarithmic and trigonometric. The process of studying the topic of trigonometric equations can be completed by studying equations involving trigonometric functions under the module symbol. When solving such equations, students are required to have thoroughly mastered the basic properties of the module and methods for solving trigonometric equations.

When solving trigonometric equations involving the modulus symbol, it is recommended to use the following general algorithm:

Step 1. Determine the intervals in which the sign of the expression under the modulus sign in the given equation does not change (i.e., find the values of *x* for which the argument of the modulus is zero).

Step 2. Remove the modulus sign in each interval: if the argument is positive — leave it as is, if negative — change the sign.

Step 3. Solve the resulting simple trigonometric equation for each interval.

Step 4. Check the obtained solutions for their inclusion in the appropriate range and isolate the corresponding ones.

Step 5. Combine the solutions from all the intervals and write the general solution.

Many students make the following mistakes when solving modular equations: 1) not taking into account the interval when solving the module - the interval should always be clearly defined; 2) $n \in Z$ forgetting that the general solution of the trigonometric equation is being written; 3) not checking for foreign roots - this mistake is especially common when squaring both sides of the equation. Therefore, it is necessary to check each solution by putting it in its original form.

Example 4. $|\sin x + \cos x| = 1$ Solve the equation.

Solution. This equation $\sin x + \cos x = 1$ and $\sin x + \cos x = -1$ are brought to solve the equations. We solve them:

1) $\sin x+\cos x=1,\ \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)=1,\ \sqrt{2}\left(\cos x\cdot\cos\frac{\pi}{4}+\sin x\cdot\sin\frac{\pi}{4}\right)=1,$

$\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\ x-\frac{\pi}{4}=\pm\frac{\pi}{4}+2\pi k,\ x=\frac{\pi}{4}\pm\frac{\pi}{4}+2\pi k,\quad k\in Z;$

2) $\sin x+\cos x=-1,\ \cos\left(x-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}},\ x-\frac{\pi}{4}=\pm\frac{3\pi}{4}+2k\pi,\ x=\frac{\pi}{4}\pm\frac{3\pi}{4}+2k\pi,\ k\in Z.$ Example 5. $|\sin x|=\sin x+2\cos x$ Solve the equation .

Solution. 1) $\sin x\geq 0$ Suppose . Then the following equation is formed from the given equation: $\sin x=\sin x+2\cos x,\ \ 2\cos x=0,\ \cos x=0,\ \ x=\frac{\pi}{2}+2k\pi\,(k\in Z)$. This $\sin x\geq 0$ satisfies the condition.

2) $\sin x<0$ Suppose . Then the following equation is formed from the given equation: $-\sin x=\sin x+2\cos x,\ -2\sin x=2\cos x,\ \sin x=-\cos x$. The only $x=-\frac{\pi}{4}+2k\pi\ (k\in Z)$ solution to this equation $\sin x<0$ satisfies the condition.

Answer: $x_1=\frac{\pi}{2}+2k\pi,\ \ x_2=-\frac{\pi}{4}+2k\pi\ (k\in Z).$

Example 6. $|tgx|=tgx-\frac{1}{\cos x}$ Solve the equation .

Solution. We write the given equation $\frac{1}{\cos x}=tgx-|tgx|$ in the form. The left side of this equation cannot be zero. Therefore, it cannot tgx be positive or zero ., i.e. $tgx<0$. Then $\frac{1}{\cos x}=tgx+tgx,\ \ \frac{1}{\cos x}=2tgx,\ \ \frac{1}{\cos x}=2\cdot\frac{\sin x}{\cos x},\qquad \frac{2\sin x-1}{\cos x}=0,$

$$\begin{cases}\sin x=\frac{1}{2},\\ \cos x\neq 0,\\ tgx<0\end{cases};\quad \begin{cases}x=\frac{\pi}{6}+2k\pi\quad yoki\quad x=\frac{5\pi}{6}+2k\pi\quad (k\in Z),\\ tgx<0\end{cases}\qquad x=\frac{5\pi}{6}+2k\pi\quad (k\in Z)\,.$$

Answer : $x=\frac{5\pi}{6}+2k\pi\quad (k\in Z)$.

Example 7. $2\cos^2 x=|\sin x|$ Solve the equation .

Solution. $\cos^2 x=1-\sin^2 x$ Since we write $|\sin x|=y$ the given equation in the form $2-2\sin^2 x=|\sin x|$ If we take , $\sin^2 x=y^2$ then we get and the given equation $2-2y^2=y$ takes the form or .

$2y^2+y-2=0$ From this $y_1=\frac{-1+\sqrt{17}}{4}$ we get $y_2=\frac{-1-\sqrt{17}}{4}$ or ($|\sin x|=\frac{-1+\sqrt{17}}{4}$ which is not in the domain of the equation). Since here , the solution of the last equation is as follows: $0<\frac{-1+\sqrt{17}}{4}<1$

$$x=(-1)^k \arcsin\frac{-1+\sqrt{17}}{4}+k\pi \ \ (k\in Z).$$

Answer: $x=(-1)^k \arcsin\frac{-1+\sqrt{17}}{4}+k\pi \ \ (k\in Z).$

Example 8. $3tgx=\sqrt{3}|\sin x|$ Solve the equation .

Solution. The period of both sides of the equation π is equal to . Therefore, its solution $[0;\pi]$ can be sought in the interval. $x_1=0$ satisfies the equation . So, it is a solution. If $x\neq 0$ so, then $\sin x\neq 0$ we get from the given equation . $\cos x=\sqrt{3}$ This equation has no solution. So, all solutions of the given equation $x=n\pi \ (n\in Z)$ are .

Answer : $x=n\pi \ (n\in Z)$.

Example 9. $2\sin^2 x=\left|\sqrt{3}tgx\right|$ Solve the equation .

Solution. Since both sides of the equation are positive, we can obtain an equation equivalent to it by squaring both sides: $4\sin^4 x=3tg^2x$. If $\sin^2 x=y$ we assume that, then $4y^2=\frac{3y}{1-y}$, $4y^2-4y^3=3y$, $y(4y^2-4y+3)=0;$ $y=0$ ($4y^2-4y+3$ the equation has no solution). Therefore, $\sin^2 x=0$ it is, and from this $x=k\pi \ (k\in Z)$.

Answer: $x=k\pi \ (k\in Z)$.

Example 10. $|\cos x|=\cos x-2\sin x$ Solve the equation .

Solution. $\cos x\geq 0$ if, $\cos x=\cos x-2\sin x,\ \sin x=0,\ x=2k\pi,\ k\in Z;$ $\cos x<0$ if, $-\cos x=\cos x-2\sin x,\ \cos x=\sin x,\ tgx=1,\ x=\frac{5\pi}{4}+2k\pi,\ k\in Z.$

Answer: $x_1=2k\pi,\ x_2=\frac{5\pi}{4}+2k\pi \ (k\in Z).$

Example 11. $\cos x=|\cos x|(x+1,5)^2$ Solve the equation .

Solution. 1) $\cos x=0$ If , then 0=0, in which case $x=\frac{\pi}{2}+k\pi \ (k\in Z).$

2) $\cos x>0$ if , $\cos x=\cos x(x+1,5)^2$, $(x+1,5)^2=1$, $x+1,5=\pm 1;$ $x_1=-0,5,$ $x_2=-2,5.$ $\cos(-0,5)=\cos 0,5>0$, because $0<0,5<\frac{\pi}{2}$; $\cos(-2,5)=\cos 2,5<0$, because $\frac{\pi}{2}<2,5<\pi$. So, $x=2,5$ it cannot be a root.

3) $\cos x < 0$ If , $-1 = (x+1,5)^2$ then , and this has no solution.

Answer: $x_1 = -0,5;\ x_2 = \frac{\pi}{2} + k\pi \ \ (k \in Z).$

Example 12. $\cos x = |\sin x|$ Solve the equation .

Solution. 1) $\sin x \geq 0$ if , $\cos x = \sin x,\ \ tgx = 1,\ \ x = \frac{\pi}{4} + 2k\pi \ \ (k \in Z)$.

2) $\sin x < 0$ if , $\cos x = -\sin x,\ \ tgx = -1,\ \ x = -\frac{\pi}{4} + 2k\pi \ (k \in Z)$.

Answer: $\frac{\pi}{4} + 2k\pi$, -$\frac{\pi}{4} + 2k\pi \quad (k \in Z)$.

Example 13. $\sqrt{3}\sin x = |\cos x|$ Solve the equation .

Solution. $\sqrt{3}\sin x = |\cos x| \Leftrightarrow \begin{cases} 3\sin^2 x = \cos^2 x, \\ \sin x \geq 0. \end{cases} \Leftrightarrow \begin{cases} 4\sin^2 x = 1, \\ \sin x \geq 0 \end{cases} \Leftrightarrow \begin{cases} \sin x = \frac{1}{2}, \\ \sin x \geq 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow x = (-1)^k \frac{\pi}{6} + k\pi \ (k \in Z).$

Answer : $x = (-1)^k \frac{\pi}{6} + k\pi \ (k \in Z)$.

Example 14. $\cos x = tgx|\cos x|$ Solve the equation .

Solution. We square both sides of the equation. $\cos^2 x = tg^2 x \cdot \cos^2 x$. Since $\cos^2 x > 0$ the domain of the equation is $\cos x \neq 0$. Therefore, from the last equation $tg^2 x = 1$ We find . From this we get $tgx = \pm 1,\ \ x_1 = \frac{\pi}{4} + k\pi,\ x = \frac{\pi}{4} + k\pi \ (k \in Z)$.

During the squaring process, extraneous roots may appear. Therefore, $\cos x$ and tgx We need to separate the roots whose signs are the same. They are: $x = \frac{\pi}{4} + 2k\pi$ and $x = \frac{3\pi}{4} + 2k\pi \ \ (k \in Z)$.

Answer: $x = \frac{\pi}{4} + 2k\pi$, $x = \frac{3\pi}{4} + 2k\pi \ \ (k \in Z)$.

Example 15. $|\sin x| + \sin x(x-4)^2 = 0$ Solve the equation.

Solution. $\sin x > 0$ If , the left side of the equation is positive and in this case the equation has no solution. $\sin x = 0$ If , then the equation $x = k\pi \ (k \in Z)$ will have a root. $\sin x < 0$ If so, then we have: $-\sin x + \sin x(x-4)^2 = 0,\ \ \sin x \cdot \left[(x-4)^2 - 1\right] = 0,\ \ (x-4)^2 - 1 = 0,\ \ (x-4)^2 = 1,\ x - 4 = \pm 1,\ \ x_1 = 5,$

$x_2 = 3$ will be. $\pi < 5 < 2\pi$ Since and $\frac{\pi}{2} < 3 < \pi$ since $\sin 3 > 0$. Therefore, x=5 is a root of the given equation. $\sin 5 < 0$

Answer: $x=5,\ x=k\pi\ (k\in Z)$.

Example 16. $4^{|x-2|\sin x}=2^{x|\sin x|}$ Solve the equation .

Solution $4^{|x-2|\sin x}=2^{x|\sin x|},\quad 2^{2|x-2|\sin x}=2^{x|\sin x|},\quad 2|x-2|\sin x=x|\sin x|$.

1) If $\sin x=0$ so, then $x=k\pi\ (k\in Z)$.

2) If $\sin x>0$ so, then $2|x-2|\sin x=x\sin x,\quad 2|x-2|=x$;

$\begin{cases}3x^2-16x+16=0,\\ x\geq 0\end{cases}$; $x=4$ or $x=\frac{4}{3}$.

$\frac{3\pi}{2}<4<2\pi$ since . Therefore $\sin 4<0$, $x=4$ the given equation cannot have a root. $0<\frac{4}{3}<\frac{\pi}{2}$ Since. Therefore $\sin\frac{4}{3}>0$, $x=\frac{4}{3}$ it is a root of the given equation.

Answer $x=k\pi\ (k\in Z),\ x=\frac{4}{3}$.

Example 17. $\sin x=tgx|\sin x|$ Solve the equation.

Solution. 1) $\sin x=0$ if , $x=k\pi\ (k\in Z)$.

2) $\sin x>0$ if, $\sin x=tgx\cdot\sin x,\quad tgx=1,\quad x=\frac{\pi}{4}+k\pi\ (k\in Z)$. This solution satisfies $\sin x>0$ the condition $x=\frac{\pi}{4}+2k\pi\ (k\in Z)$.

3) $\sin x<0$ if , $\sin x=-tgx\cdot\sin x,\quad tgx=-1,\quad x_{1,2}=\pm\frac{\pi}{4}+2k\pi$.

Answer : $x_{1,2}=\pm\frac{\pi}{4}+2k\pi$, $x_3=k\pi\ (k\in Z)$.

examines in detail trigonometric equations involving the modulus sign and methods for solving them. The concept of the modulus of a real number is explained using three definitions — analytical, geometric, and algebraic — and its six main properties are stated without proof. The basic rules for opening the modulus sign: | *f(x)* | = *a* equation *f(x)* = *a* and It was shown that the equation of the form | *f(x)* | = g(x) can be replaced by a set of equations *f(x)* = *−a ; and the equation of the form | f(x)* | = *g(x)* can be replaced by two systems.

The article presents 17 examples of varying difficulty levels, from simple modular equations to more complex equations that use various solution methods, such as substitution of variables, squaring, and the method of intervals. These examples will greatly help students master the topic step by step.

From a pedagogical point of view, it is advisable to adhere to the following methodological recommendations when teaching trigonometric equations involving the modulus symbol: first, to visually explain the geometric meaning of the modulus using a number line; second, to present the solution algorithm to students in the form of a table or diagram; third, to reinforce each new method first with a simple example, and then with a complex example; fourth, to form in students the habit of checking the solutions obtained.

In the future, to further expand this topic, it is recommended to study trigonometric inequalities involving the modulus sign, as well as complex equations involving two or more modulus signs. Such equations are often found in DTM exams and international mathematics olympiads, so studying them is of practical importance for students.

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