European International Journal of Pedagogics (ISSN: 2751-000X)

Received: 31 December 2025 | Accepted: 22 January 2026 | Published: 28 February 2026

DOI: 10.55640/eijp-06-02-36

RESEARCH ARTICLE

A Didactic Model for Forming Mathematical Competence in Primary Education Through Geometric Concepts

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VOLUME: Vol.06 Issue02 2026

PAGE: 167-171

Abstract

The competence-based transformation of primary mathematics education requires instructional models that connect conceptual understanding, procedural fluency, reasoning, and communication within meaningful learning situations. Geometry provides a particularly powerful pathway for competence formation because it naturally integrates visualisation, measurement, spatial reasoning, and argumentation, while also supporting the development of mathematical language. This article proposes and justifies a didactic model for forming mathematical competence in primary education through fundamental geometric concepts. The model is grounded in documentary analysis of international frameworks that emphasise mathematical reasoning and application, including the cognitive domain logic of TIMSS (knowing, applying, reasoning) and the mathematical literacy orientation of PISA, alongside research on developmental progressions in geometric thinking (notably the van Hiele theory). The model is described as a coherent system linking learning outcomes, content selection, instructional design, formative assessment, and feedback loops. It specifies how core geometric ideas—point, line, segment, ray, angle, plane figures, symmetry, perimeter, area, and spatial relationships—can function as "conceptual organisers" for broader mathematical competence. The proposed design foregrounds representational transitions (from concrete manipulation to schematic drawings and symbolic notation), discourse routines, and criterion-referenced assessment. The article argues that the model is feasible for teacher education and school practice because it offers operational design principles for lesson planning, task construction, and assessment. It also defines evaluation indicators for monitoring competence growth, including shifts in students' reasoning quality, use of geometric language, and ability to model real situations with geometric representations.

KEYWORDS

Primary mathematics education, mathematical competence, geometry, didactic model, van Hiele levels, formative assessment, mathematical literacy, TIMSS cognitive domains, PISA framework, spatial reasoning.

INTRODUCTION

In contemporary primary education, "mathematical achievement" is increasingly understood as more than correct answers on routine exercises. Education systems seek learners who can interpret situations, choose representations, reason, communicate, and apply mathematical ideas flexibly. This shift is consistent with international assessment frameworks that define competence in terms of reasoning and application rather than reproduction of procedures. The TIMSS framework explicitly distinguishes cognitive expectations as knowing, applying, and reasoning, emphasising that content mastery should be accompanied by meaningful use of knowledge in tasks that require understanding and justification. PISA similarly conceptualises mathematical literacy as the capacity to reason mathematically and to formulate, employ, and interpret mathematics in real-world contexts.

Within this competence-oriented agenda, geometry occupies a strategic role in primary mathematics. Fundamental geometric concepts support the formation of spatial representations, the development of mathematical language, and the emergence of early deductive reasoning. Geometry also provides a natural bridge between mathematics and everyday experiences because children encounter shapes, patterns, symmetry, measurement, and spatial relations in their environment long before they meet formal symbolic mathematics. When geometry is treated as an isolated unit focused on naming shapes, its potential remains underused. When it functions as a conceptual backbone for wider mathematical activity—connecting number, measurement, data representation, and problem solving—it becomes a resource for competence formation.

At the same time, research consistently shows that geometric thinking is developmentally structured and that many learners and even pre-service teachers remain at relatively low levels of geometric reasoning unless instruction is deliberately designed to support progression. The van Hiele theory describes levels of geometric thinking that move from visual recognition to analysis of properties, then to informal deduction and beyond. Studies in primary contexts indicate that deficiencies in geometric thinking levels appear across countries and curricula, suggesting that the problem is not local but structural, tied to how geometry is taught and assessed. This makes a didactic model necessary: teachers need a coherent design that translates broad competence aims into teachable sequences, tasks, representations, and assessment practices.

The purpose of this article is to propose a didactic model for forming mathematical competence in primary education through geometric concepts and to justify its theoretical basis using international frameworks and research on geometric cognition. The guiding questions are: How can fundamental geometric concepts be organised so they support broader mathematical competence rather than remain a separate topic? How should instruction be structured so that students move from visual intuition toward property-based reasoning and argumentation? What assessment logic can capture competence growth in ways that inform teaching?

This study employs a design-oriented documentary analysis and conceptual modelling approach. First, key constructs were extracted from international frameworks that articulate competence expectations in mathematics. TIMSS materials were analysed for their content emphases and cognitive domain structure, with particular attention to the positioning of measurement and geometry in grade 4 mathematics and the definition of knowing, applying, and reasoning as intended cognitive processes. PISA documents were analysed to clarify the definition of mathematical literacy and its emphasis on reasoning, modelling, and interpretation as central dimensions of competence.

Second, research literature on geometric thinking was reviewed to identify developmental progressions relevant to primary instruction. The van Hiele model was used as a theoretical lens for sequencing learning experiences and predicting typical difficulties, drawing on widely cited explanations of the levels and evidence that learners often remain at lower levels without targeted instruction. This theoretical grounding was complemented by sources discussing how van Hiele phases of learning can influence outcomes in geometry education.

Third, the didactic model was constructed as an integrative system that connects five elements: competence outcomes, content and concept selection, instructional design principles (including representational transitions and discourse), formative assessment and feedback, and evaluation indicators for monitoring growth. The model is presented in a form that can be applied both in primary classroom practice and in teacher education programs as a planning and reflection tool.

The proposed didactic model treats geometric concepts as organisers of mathematical competence rather than as a narrow content strand. In the model, competence is operationalised as the student's ability to recognise and create geometric representations, interpret and use properties, perform measurements meaningfully, reason about relationships, and communicate solutions using appropriate mathematical language. This competence definition aligns with international frameworks that highlight reasoning and application as core outcomes rather than optional enrichment.

The model is built around a coherent "conceptual corridor" that begins with basic geometric primitives and develops toward relational reasoning. At the entry point, learners work with point, line, segment, ray, angle, and basic plane figures. These are not introduced primarily as vocabulary items but as tools for describing space and action: drawing paths, marking positions, describing turns, comparing lengths, and constructing shapes. From this base, the corridor extends into symmetry and transformations, perimeter and area as measures linked to structure, and spatial relations such as parallelism, perpendicularity, and decomposition–composition. In this organisation, number sense and arithmetic are continuously connected to geometric meaning through measurement, counting units, and comparing quantities in spatial contexts.

A defining feature of the model is its alignment between content and cognitive demand. The knowing dimension is addressed through accurate recognition, naming, and basic property awareness. The applying dimension is supported through modelling tasks where learners use geometric ideas to solve practical problems, such as planning a route, determining required materials by measuring perimeters, or choosing shapes that satisfy constraints. The reasoning dimension emerges when learners justify classifications, explain why a construction works, compare strategies, or generalise patterns in shapes and measurements. This tripartite structure is consistent with the TIMSS cognitive domain logic and helps teachers design a balanced learning trajectory rather than over-concentrating on recognition-level activities.

The model embeds a developmental progression inspired by the van Hiele theory. Early instruction assumes that many learners start from a visual level in which shapes are recognised by appearance and prototypes. The model therefore emphasises tasks that shift attention from appearance to properties by inviting comparison, sorting, and describing differences using measurable or observable attributes. As learners move toward an analytic level, they begin to articulate properties such as equal sides, equal angles, symmetry, and parallel sides. This progression is essential because the ability to reason in geometry depends on property-based thinking rather than on visual impression alone.

Crucially, the model does not treat progression as automatic. It incorporates deliberate representational transitions, starting with concrete manipulation and embodied actions (folding, tracing, building, rotating), moving toward schematic drawings and grid representations, and gradually introducing symbolic notations and conventional measurement tools. Such transitions are designed to support abstraction while protecting meaning. Research indicating that many learners show limited acquisition of higher van Hiele levels supports the need for structured phases of learning rather than incidental exposure.

The model specifies how instruction can operationalise competence formation through three mutually reinforcing mechanisms: task design, discourse design, and feedback design.

Task design is guided by the idea that a geometric concept becomes competence-building when it functions as a tool for solving or explaining. Tasks are therefore structured to require representation choice, property use, or measurement interpretation. For example, when teaching rectangles and squares, the task emphasis shifts from naming to reasoning about necessary and sufficient properties, and to applying these properties in construction activities under constraints. When teaching symmetry, tasks require learners to predict outcomes of reflections or to verify symmetry by argument, not only to identify "beautiful patterns."

Discourse design is treated as a core didactic tool. Competence in mathematics is partly competence in language: being able to state definitions informally, explain a procedure, justify a claim, and interpret others' reasoning. The model therefore incorporates classroom routines in which learners describe shapes using property language, compare solutions, and respond to prompts that require justification. This connects directly to the reasoning emphasis in international frameworks, where the ability to reason and interpret is central to competence.

Feedback design is formative and criterion-referenced. The model assumes that feedback is effective when it points learners to the next step in reasoning or representation rather than merely marking correctness. In geometry, this often means feedback on the adequacy of property use, the precision of measurement, and the clarity of explanation. Students learn that a correct drawing without justification may be incomplete when the goal is reasoning, whereas a partially correct result with strong explanation may demonstrate deeper competence growth.

The model proposes an assessment logic aligned with competence. Evidence is gathered from student work in multiple forms: drawings and constructions, measurement records, written explanations, and oral reasoning. Instead of treating assessment as a separate event, the model integrates assessment into learning cycles so that tasks generate evidence and evidence informs subsequent instruction.

Competence growth is evaluated by observing shifts in the quality of representations and explanations. Early indicators include more accurate use of geometric vocabulary and improved precision in constructing and measuring. Intermediate indicators include the ability to classify shapes by properties, to identify invariants under transformations, and to connect measurements to structure. Advanced indicators at the primary level include emerging argumentation: explaining why a shape belongs to a class, justifying a strategy, or generalising a pattern. This indicator system supports teachers in moving beyond score-based monitoring toward evidence-based pedagogical decision making.

The proposed model responds to a recurring challenge in primary mathematics: geometry is often taught as a set of labels, while competence aims demand reasoning, application, and communication. By treating geometric concepts as organisers, the model creates coherence between content and competence. This coherence is compatible with TIMSS's framing of cognitive domains and the significant share of grade 4 mathematics devoted to measurement and geometry, which underscores the global importance of this domain in primary achievement expectations. It also aligns with PISA's emphasis on reasoning and real-world problem solving, because geometry provides natural contexts for modelling and interpreting spatial situations.

A key implication concerns teacher preparation. Many primary teachers have stronger confidence in arithmetic than in geometry, and pre-service teachers may hold procedural or visual-only conceptions of geometry. Research that assesses van Hiele levels among elementary pre-service teachers points to the need for deliberate development of geometric thinking within teacher education. The model can function as both an instructional guide and a professional learning framework: it helps future teachers design tasks with explicit cognitive intent, anticipate typical misconceptions, and interpret student work developmentally.

The model also mitigates a common risk in competence-based reforms: replacing traditional exercises with "creative activities" that are engaging but weakly aligned to learning goals. In the proposed design, innovation is not defined by novelty of activity but by the alignment between concept, cognitive demand, and evidence. This alignment protects learning depth and supports fairness, because assessment criteria remain transparent and linked to the intended competence outcomes.

At the same time, implementation constraints should be acknowledged. The model requires teacher capacity to orchestrate discourse, diagnose reasoning, and provide criterion-referenced feedback. It also benefits from access to simple manipulatives and measurement tools, though it does not depend on expensive technologies. The model can be adapted to different curricula by mapping local standards onto the conceptual corridor and ensuring that tasks cover required content while maintaining a balance of knowing, applying, and reasoning.

This article proposed a didactic model for forming mathematical competence in primary education through geometric concepts. Grounded in international competence expectations and research on developmental progressions in geometric thinking, the model frames geometry as a conceptual organiser that integrates representation, measurement, reasoning, and communication. Its main contribution is a coherent design logic linking learning outcomes, concept selection, instructional mechanisms, formative assessment, and evaluation indicators. The model supports both classroom practice and teacher education by providing an operational framework for lesson design and evidence-based reflection. Future work can validate the model empirically through classroom interventions and comparative studies examining its impact on students' reasoning quality, mathematical language development, and transfer to non-geometric problem contexts.

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