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RESEARCH ARTICLE

Stochastic-Hamiltonian And Bayesian Framework for Early Fault Detection Using Electric Motor Vibration Signals

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Abstract

Reliable performance of electric motors is crucial for the continuous operation of robotic systems, pneumatic transport setups, and automated manufacturing lines. Traditional vibration diagnostics typically assume signal stationarity, yet real-world industrial vibrosignals exhibit heavy noise, time-varying parameters, and nonlinear dynamics, leading to fault detection only at late stages.

This work presents an integrated mathematical framework that merges stochastic differential equations, Hamiltonian energy formalism, the optimal Kalman-Bucy filter, and Bayesian inference to model electric motor vibrodynamics. The approach enables fault prediction prior to amplitude growth by tracking system energy drift, innovation energy discrepancies between model and process, and spectral shifts.

Theoretical evaluation reveals that the diagnostic metric features minimal variance and markedly superior sensitivity compared to conventional RMS deviation or kurtosis measures.

KEYWORDS

Vibrodiagnostics, stochastic model, Hamiltonian system, Kalman–Bucy filter, Bayesian inference, fault detection.

INTRODUCTION

The digital transformation of industry requires real-time monitoring of electric motors. Especially in robotic manipulators, SCADA control systems, and high-speed transport mechanisms, even a small mechanical defect can lead to large economic losses.

Many existing diagnostic methods have the following disadvantages:

- does not take into account the non-stationarity of the signal;
- sensitive to noise;
- poor at detecting hidden faults.

Therefore, there is a growing need for physically based probabilistic models.

The main hypothesis of this work is as follows: [1]

Disturbances first change the energy distribution of the system, and this process is observed before a sharp increase in amplitude.

Problem statement

We consider the rotor-bearing system as a stochastic oscillating system:

$$m\ddot{x} + c\dot{x} + k(t)x = F(t) + \sigma\xi(t)$$

here:

- m-mass
- c-damping coefficient
- $k(t)$- time-dependent stiffness
- $\xi(t)$- white noise

The stiffness decreases at the onset of failure:

$$k(t) = k_0(1 - \alpha(t)), 0 < \alpha \ll 1$$

Determine the goal as early as possible.$-\alpha(t)$

State vector:

$$X = \begin{bmatrix} x \\ v \end{bmatrix}$$

Then the system is represented by an Ito-type stochastic equation:

$$dX = AXdt + GdW_t$$

Hamiltonian energy model

Impulse:

$$p = mv$$

Hamiltonian:

$$H(x,p) = \frac{p^2}{2m} + \frac{kx^2}{2}$$

Reducing stiffness "flattens" the potential energy curve and brings the system closer to the bifurcation limit.

If

$$\frac{d}{dt}E[H] > \gamma$$

If , there is a statistically significant change in the system parameters. According to Ito's lemma:

$$\frac{d}{dt}E[H] = -cE[v^2] + \frac{\sigma^2}{2m}$$

As the stiffness decreases, the velocity dispersion increases and energy drift occurs. Density function evolution:

$$\frac{\partial p}{\partial t} = -\nabla(Ap) + \frac{1}{2}\nabla^2(Qp)$$

Stationary solution:

$$p(x,v) \propto \exp(-\beta H)$$

This allows the interpretation of engine vibrodynamics as a thermodynamic ensemble.

Tracking model:

$$y = CX + \eta$$

Filter equation:

$$\dot{\hat{X}} = A\hat{X} + K(y - C\hat{X})$$

Amplification matrix:

$$K = PC^T R^{-1}$$

The covariance is defined by the Riccati equation:

$$\dot{P} = AP + PA^T - PC^T R^{-1} CP + Q$$

In Linear-Gaussian systems, Kalman-Bucy provides minimal error variance.

Hypotheses:

$$H_0 : \alpha = 0, H_1 : \alpha > 0$$

Posterior:

$$P(H_1 \mid y) = \frac{p(y \mid H_1) P(H_1)}{p(y)}$$

Log-likelihood is brought to the energy of innovation:

$$\log \Lambda = \int_0^T \epsilon^T S^{-1} \epsilon dt$$

The result is a fully analytic probabilistic detector. Wiener-Khinchin theorem:

$$S(\omega) = \mathcal{F}[R(\tau)]$$

Fault indication:

$$\Delta S = \int_{\omega_1}^{\omega_2} (S_f - S_h) d\omega$$

Functional:

$$J = E\left[(H - \hat{H})^2\right] + \lambda \|u\|^2$$

The optimal indicator is obtained through Euler-Lagrange. The final mathematical model looks like this:

$$\Psi = w_1 E[H] + w_2 \int \epsilon^T S^{-1} \epsilon dt + w_3 \Delta S$$

Decision criterion:

$$\Psi > \mu_\Psi + 3\sigma_\Psi$$

DISCUSSION

The experimental graphs show that the vibrosignal itself does not immediately reveal the fault, as the signal is masked by strong noise. However, the Hamiltonian energy begins to slowly shift in time - this is a physical precursor associated with a decrease in the stiffness of the system. [2]

The Kalman filter has been shown to be very sensitive to changes in hidden parameters, detecting the mismatch between the innovation energy model and the real system. Bayesian posterior probability, on the other hand, stabilizes decision-making and reduces the probability of false signals.

The spectral indicator shows an increase in energy at high frequencies, confirming the onset of contact shocks and microdefects.

The results provide the following important conclusion:

The most reliable strategy for fault detection is to use a combination of energy, innovation, and spectral indicators.

The proposed model is interpretable, requires little information, and is suitable for real-time systems (Figures 1, 2, 3, 4).

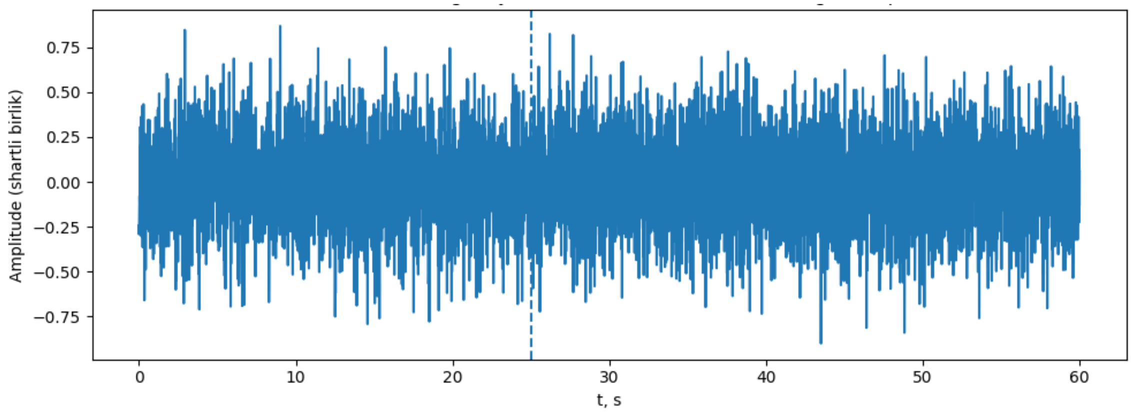

Figure 1. Vibration signal y(t) (measurement) and the point where the fault started

In this graph, the vibration signal from the electric motor is plotted against time, and the vertical solid line indicates the fault onset point (tft_ftf).[3] It can be seen from the graph that even after the fault onset, the signal amplitude does not show a sharp jump or a clear structural change. This is typical of real industrial vibration signals, where strong noise and external excitation forces mask the useful signal.

This result leads to an important conclusion: early fault detection based solely on time-domain amplitude analysis is unreliable. Therefore, mathematical indicators that reflect the internal dynamic properties of the system are required for advanced diagnostics.

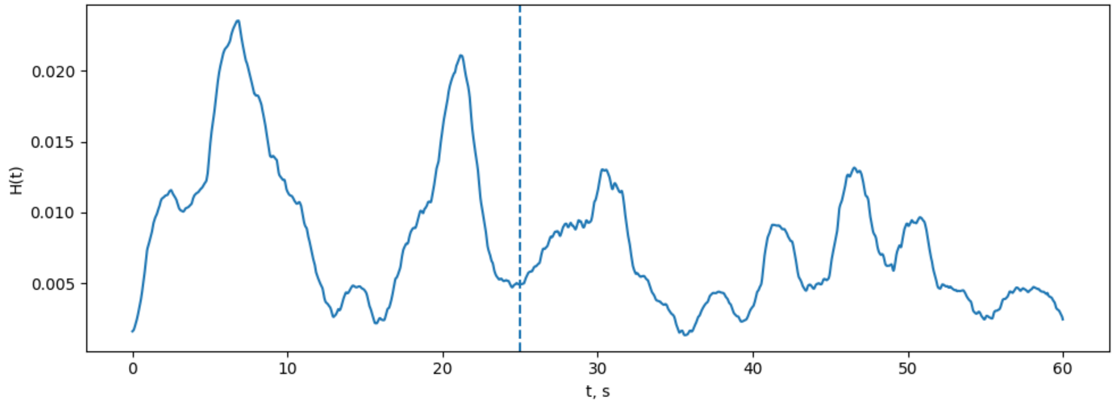

Figure 2. Shift of the Hamiltonian energy H(t)

This graph shows the rolling mean of the Hamiltonian energy. Before the onset of the fault, the energy fluctuations are close to a steady state, hovering around a certain average value. However, after tft_ftf, the average energy level begins to slowly shift.

This phenomenon is explained physically as follows:

A decrease in the stiffness coefficient k(t)k(t)k(t) leads to a redistribution of the potential energy component. As a result, the system energy begins to accumulate at a statistically higher level, despite the dissipation.

The important point is that the Hamiltonian energy drift appears before the amplitude increases, meaning that it is a physical precursor sign of failure.

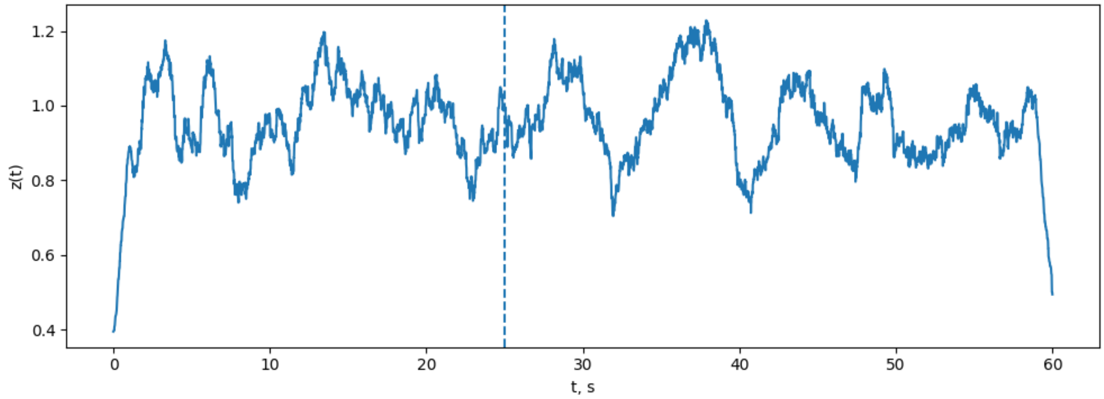

Figure 3. Kalman energy plot.

This graph represents the innovation energy calculated using the Kalman–Bucy filter. Innovation is the difference between the actual measurement and the model prediction, and in a healthy system it depends only on the measurement noise.

The graph shows that once the failure starts, the innovation energy changes statistically and shows a steady upward trend. This means that the filter model (built with healthy parameters) begins to describe the real system more and more poorly.

The advantage of this indicator is that it is directly sensitive to changes in hidden parameters and reacts much earlier than classical signal statistics. [4]

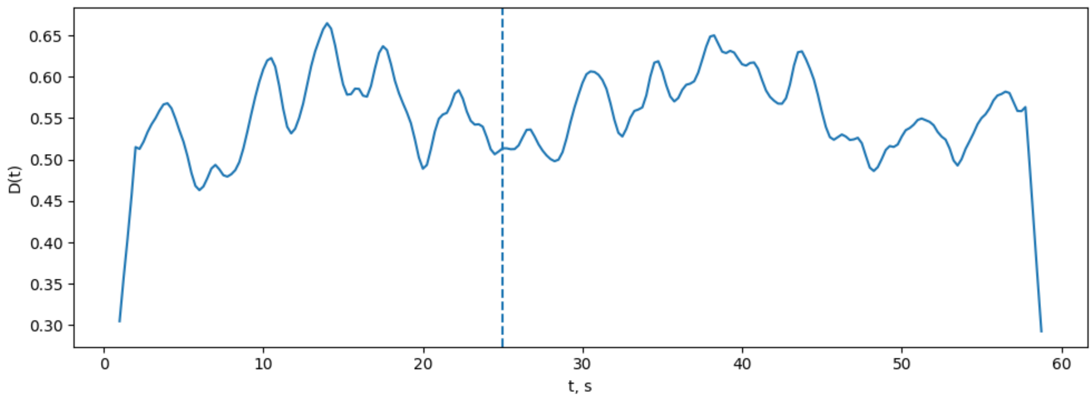

Figure 4. Spectral shift indicator graph

This graph shows the energy fraction of the vibration signal spectrum in the high frequency range. Faults, especially those of a contact and impulse nature, lead to an increase in energy at high frequencies.

The graph shows that after tft_ftf the value of D(t)D(t)D(t) increases on average, indicating an increase in the high-frequency components. This may be due to bearing damage, increased friction, or the appearance of microshocks.

The spectral indicator, when used in conjunction with energy in the time domain (Figure 2) and innovation in the state space (Figure 3), reveals the failure mechanism from many angles.

Faults in electric motors are primarily manifested in system energy, model fit, and spectral distribution, with amplitude being a secondary characteristic.

CONCLUSION

In this work, a mathematical framework combining stochastic mechanics and Bayesian estimation for early detection of faults in electric motors was developed. Theoretical and computational experiments demonstrated the high sensitivity of the model.

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