Next Scientists
European International Journal of Multidisciplinary Research and Management Studies

TYPE Original Research
PAGE NO. 19-23
DOI 10.55640/eijmrms-05-04-04

Check for updates

Filtration of Suspensions with The Formation of a Nonlinearly Compressible Sedimentary Layer

Parmonov J.T.

Samarkand State University of Architecture and Civil Engineering named after Mirzo Ulugbek, Jizzakh State Pedagogical University, Uzbekistan

Nishanov.U.A.

Samarkand State University of Architecture and Civil Engineering named after Mirzo Ulugbek, Jizzakh State Pedagogical University, Uzbekistan

Nurmatov K.J

Samarkand State University of Architecture and Civil Engineering named after Mirzo Ulugbek, Jizzakh State Pedagogical University, Uzbekistan

OPEN ACCESS

SUBMITED 09 February 2025
ACCEPTED 12 March 2025
PUBLISHED 08 April 2025
VOLUME Vol.05 Issue04 2025

Abstract: The article considers the problem of filtration of suspensions with the formation of a nonlinearly compressible sedimentary layer. Based on numerical calculations, the dependence of filtration characteristics on nonlinear effects is established.

Keywords: Suspension, filter, moving boundary, filter layer, sediment, pressure.

Introduction: Filtration of suspensions refers to complex technological processes and depends on a large number of micro- and macrofactors. Models of filtration of suspensions are based on fundamental equations of mechanics of multiphase, multicomponent media and the theory of filtration consolidation [1,2,3]. The problem of pressure distribution in the sediment layer is reduced to the solution of the classical parabolic equation with moving boundary conditions. Real physical conditions during filtration of suspensions are such that the sediment is heterogeneous in its structure and filtration-capacitive properties and consolidates under the action of pressure and other forces. Thus, in the general case, the consolidation coefficient is variable, depending on the pressure distribution over the sediment thickness. In this paper, the process of sedimentation during filtration of suspensions is analyzed taking into account the variability of the consolidation coefficient.

Let there be a flat filter element. The suspension is placed in such a way that it contacts the initially existing filter layer of some thickness. Under the action of the pressure difference, which remains constant over time, the process of separation of the suspension begins, the thickness of the sediment continuously increases due to the movement of the sediment - suspension boundary. Thus, two regions can be distinguished- the filter layer region, and - the sediment region, where is the moving boundary. Due to the listed conditions, the Stefan problem can be formulated for the filtration process. Let be the pore pressure, - the liquid pressure at the initial moment of time, at the entrance to the sediment layer, at the exit from the filter layer.

The mathematical model of filtration is written as follows [1,2]

$$\frac{\partial p}{\partial t} = b(p)\frac{\partial^2 p}{\partial z^2}, \quad 0 < t \le T \quad, \ z \in \Omega_2, \tag{1}$$

$$p(z,0) = p_0(z), \quad z \in \Omega_1, \tag{2}$$

$$p(0,t) = p_2 \qquad 0 < t \le T, \tag{3}$$

$$p[h(t),t] = p_1, \qquad 0 < t \le T, \tag{4}$$

$$\frac{\partial p}{\partial z} = l\frac{dh}{dt}, \qquad 0 < t \le T, \tag{5}$$

Where $h(0) = z_0$, $b(p) = b_0 e^{\alpha(p-p_1)}$ - consolidation ratio, $p_0 = p_2 + \frac{z(p_1 - p_2)}{z_0}$, $0 \le z \le z_0$ $l = \frac{r\mu}{u}$, r - sediment resistivity, μ - viscosity coefficient, u - external sedimentation coefficient, α - parameter.

Introducing a new variable

$$v = \int_0^p \frac{d\xi}{b(\xi)} = a\left(e^{-\alpha p} - 1\right), \qquad a = \frac{1}{-\alpha b_0 e^{-\alpha p_1}}$$

we will receive

$$p = -\frac{1}{\alpha}\ln\left(\frac{v}{a} + 1\right). \tag{6}$$

Then equation (1) is transformed to the form

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(k(v)\frac{\partial v(p)}{\partial z}\right), \ k(v) = -\frac{1}{\alpha(v+a)}. \tag{7}$$

After transforming conditions (2) - (5) we arrive at

$$v|_{t=0} = v_0(z) = a\left(e^{-\alpha p_0(z)} - 1\right), \tag{8}$$

$$v|_{z=0} = v_2 = a\left(e^{-\alpha p_2} - 1\right), \tag{9}$$

$$v|_{z=h(t)} = v_1 = a\left(e^{-\alpha p_1} - 1\right), \tag{10}$$

$$k(v)\frac{\partial v(p)}{\partial z} = l\frac{dh}{dt}. \tag{11}$$

To solve problem (7)-(11) we use the finite difference method [4].

Let's introduce a uniform grid z with a step f $\overline{\omega} = \overline{\omega}_1 + \overline{\omega}_2 = \{z_i \mid z_i = if,\ i = 0,1,\dots,m+j\}$, where in time we will use a non-uniform grid [4]

$$\overline{\omega}_\tau = \{t \mid t = t_j = t_{j-1} + \tau_j, t_m = 0, j = m+1, m+2, \dots, N, t_N = T\}$$

with variable pitch $\tau_j > 0$. The time interval step should be selected $[t_j, t_{j+1}]$ so that the moving boundary moves exactly one step along the spatial grid. This approach is known as the grid node front catching method [4]. For the boundary node of the dynamic stack we use the notation $z = f(t_j) = i_j f$.

We approximate equation (7) with a purely implicit scheme

$$\frac{v_i^{j+1} - v_i^j}{\tau} = \frac{1}{f}\left(\chi_{i+1}\frac{v_{i+1}^{j+1} - v_i^{j+1}}{f} - \chi_i\frac{v_i^{j+1} - v_{i-1}^{j+1}}{f}\right), \qquad i = 1,2,\dots N-1, \tag{12}$$

где $\chi_i = 0{,}5\left[k(v_i^j) + k(v_{i-1}^j)\right]$.

Approximation of the initial and boundary conditions (8)-(10) gives

$$v_i^0 = a\exp(\alpha p_i^0 - 1), \quad p_i^0 = p_2 + \frac{if(p_1 - p_2)}{mf}, \qquad i = 0,1,\dots m, \tag{13}$$

$$v_0^{j+1} = v_2, \tag{14}$$

$$v_i^{j+1} = v_1, \qquad z = i_{j+1}h_1. \tag{15}$$

Condition (11) taking into account $\frac{dh}{dt} \approx \frac{f}{\tau_{j+1}}$ we discretize it like this

$$\frac{1}{\alpha(v_i^{j+1} + a)}\frac{v_i^{j+1} - v_{i-1}^{j+1}}{f} + l\frac{f}{\tau_{j+1}} = 0, \quad z = i_{j+1}f. \tag{16}$$

Equation (12) is reduced to the form

$$A_i v_{i-1}^{j+1} - C_i v_i^{j+1} + B_i v_{i+1}^{j+1} = -F_i, \quad i = \overline{1, j}, \tag{17}$$

where

$$A_i = \frac{\chi_i \tau}{f^2}, \quad B_i = \frac{\chi_{i+1}\tau}{f^2}, \quad C_i = 1 + \frac{\chi_i \tau}{f^2} + \frac{\chi_{i+1}\tau}{f^2}, \quad F_i = v_i^j.$$

To solve the nonlinear difference problem (12) - (16) at each new time layer $t = t_{j+1}$ iterative processes can be used. The simplest of these is associated with iterative refinement of the time step τ_{j+1} [4]. Let the initial approximation be given τ_{j+1}^{0}. Given τ_{j+1}^{s} the corresponding approximation for v_i^{j+1} we find their solutions to the following nonlinear difference problem

$$A_i v_{i-1}^{s,j+1} - C_i v_i^{s,j+1} + B_i v_{i+1}^{s,j+1} = -F_i \ , \quad i = \overline{1, j} \ , \tag{18}$$

$$v_0^{s,j+1} = v_2 , \tag{19}$$

$$v_i^{s,j+1} = v_1 , \qquad \text{в узел} \ \ z_i = i_{j+1} f \ . \tag{20}$$

To solve this problem we use a three-point sweep. Relation (16) is used to determine the time step and in the simplest case we have [4]

$$\tau_{j+1}^{s+1} = lf \left(\frac{1}{\alpha \left(v_i^{j+1} + a \right)} \frac{v_{i-1}^{j+1} - v_i^{j+1}}{f} \right)^{-1} , \quad \text{в узле} \ \ z_i = i_{j+1} f \ . \tag{21}$$

Now, to solve the difference problem (18) - (20), we use the counter sweep method [3]. Here $i = m, \ 0 < m < h(t)$ - internal node. In the area $0 \le i \le m$ the solution is calculated using the right-hand sweep formulas

$$v_i^{s,j+1} = \alpha_{i+1} v_{i+1}^{s,j+1} + \beta_{i+1} \qquad i = m-1, m-2, \ldots, 1, 0 \ , \tag{21}$$

где

$$\alpha_{i+1} = \frac{B_i}{C_i - A_i \alpha_i} \ , \quad i = 1, 2, \ldots, m-1 \ , \qquad \alpha_1 = 0 \ , \tag{22}$$

$$\beta_{i+1} = \frac{F_i + A_i B_i}{C_i - A_i \alpha_i} \ , \quad i = 1, 2, \ldots, m-1 \ , \qquad \beta_1 = v_2 , \tag{23}$$

and in the region $m < i < h(t)$ - according to the left-hand sweep formulas

$$v_{i+1}^{s,j+1} = \xi_{i+1} v_i^{s,j+1} + \eta_{i+1} , \qquad i = m, m+1, \ldots, N-1 , \tag{24}$$

where

$$\xi_i = \frac{A_i}{C_i - B_i \xi_{i+1}} \ , \qquad i = N-1, N-2, \ldots, m , \qquad \xi_N = 0 , \tag{25}$$

$$\mu_i = \frac{F_i + B_i \eta_{i+1}}{C_i - B_i \xi_{i+1}} , \qquad i = N-1, N-2, \ldots, m , \qquad \eta_N = v_0 . \tag{26}$$

Considering (21), (24) in the node $i = m$ we find

$$v_m^{s,j+1} = \frac{\eta_m + \xi_m \beta_m}{1 - \xi_m \alpha_m} \ . \tag{27}$$

Based on the obtained numerical results, graphs of the moving boundary were constructed. $h(t)$, pressure distribution in the filter and sediment layer (Fig. 1-4).

From the presented graphs, one can estimate the growth of the sediment layer and the pressure distribution in it.

Growing importance α contributes to a corresponding change in the consolidation coefficient, which in turn affects the growth of the sediment layer and the pressure distribution in it. With an increase in the value α lagging growth dynamics can be observed $h(t)$. The graphs also show the dynamics of pressure at fixed points of the sediment layer. As the sediment layer grows, the pressure at fixed points decreases.

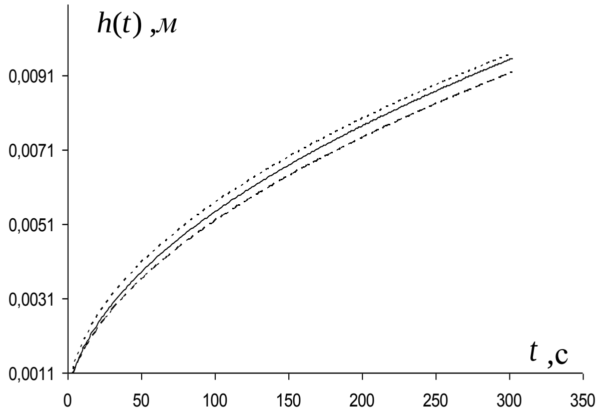

Fig. 1. Change in the thickness of the filter layer at different $\alpha = 10^{-6}$ Pa^{-1}; $\alpha = 10^{-7}$ Pa^{-1}; $\alpha = 10^{-8}$ Pa^{-1}. $t = 350$ s.

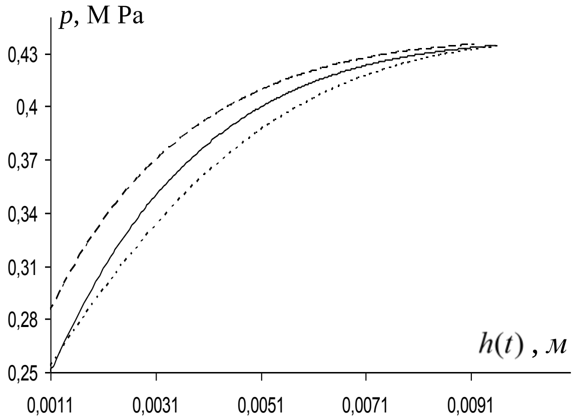

Fig. 2. Distribution of pressure across the sediment thickness at different $\alpha = 10^{-6}$ Pa^{-1}; $\alpha = 10^{-7}$ Pa^{-1}; $\alpha = 10^{-8}$ Pa^{-1} $t = 350$ s.

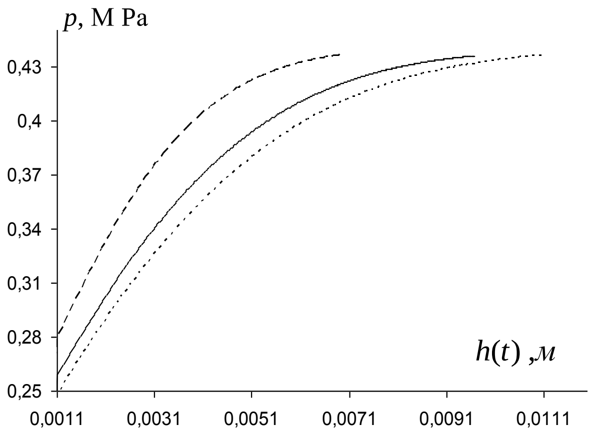

Рис 3. Distribution of pressure across the sediment thickness. $\alpha = 10^{-6}$ Pa. $t = 150$ s; $t = 300$s; $t = 400$ s

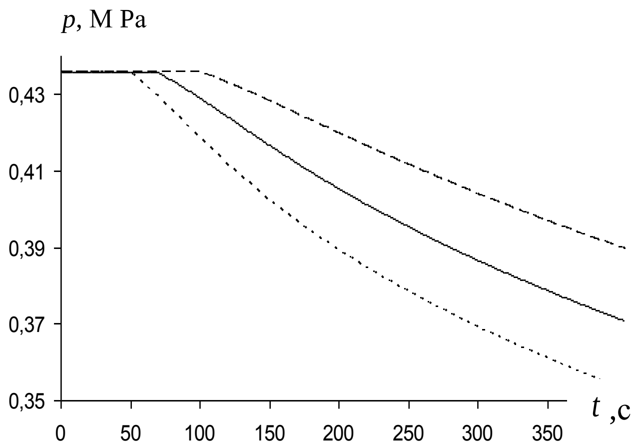

Fig. 4. Pressure distribution at fixed points of the sediment layer $\alpha = 10^{-6}$ Pa. $z = 4{,}03 \cdot 10^{-3}$ м); $z = 4{,}75 \cdot 10^{-3}$ м $z = 5{,}65 \cdot 10^{-3}$ м

REFERENCES

Федоткин И.М., Воробьев Е.И., Вьюн В.И. Гидродинамическая теория фильтрования суспензией. Киев:Вища. шк.,Головное изд-во.1986. -166с

Федоткин И.М. Математическое моделирование технологических процессов. Киев: Вища шк., Головные изд-во. 1988.-415с.

Нигматулин Р.И. Динамика многофазных сред. Т. II. М., Наука, 1987.-389с.

Самарский А.А., Вабищевич П.Н. Вычислительная теплопередача. –М.: Едиториал УРСС, 2003-784 с.

Самарский А.А. Теория разностных схем. – М.: Наука, 1977. -656с.